Davidson Institute

Fellows Scholarship

An Adaptive Hybrid Quantum Algorithm for the Metric Traveling Salesman Problem

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Problem Statement and Example

Metric Traveling Salesman Problem (Metric TSP)

- Given a **complete graph** with positive **edge weights** that satisfies the **properties of a metric space**, find a cycle starting from vertex ***v*** that visits each vertex once with the **minimal edge weight sum**.

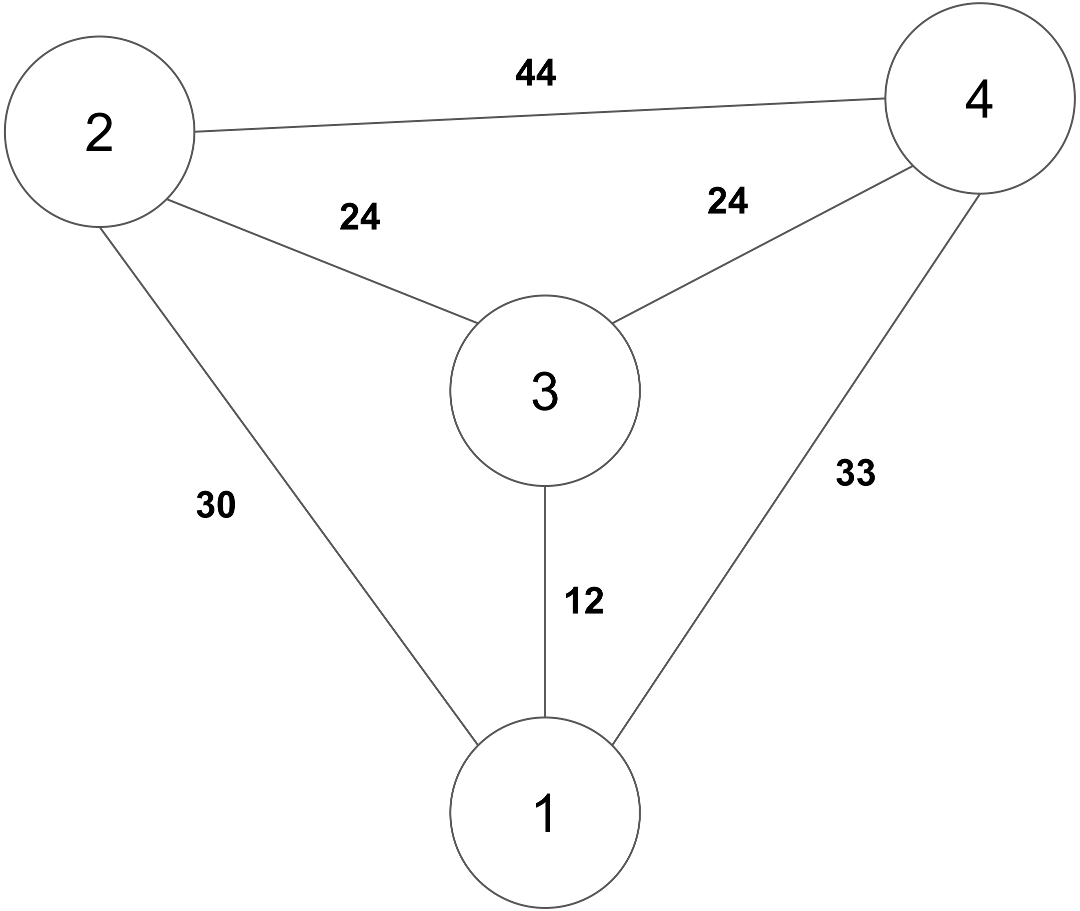
- **complete graph:** a simple undirected graph in which every pair of distinct vertices is connected by a unique edge

- **edge weight:** the value associated with an edge in the graph

- **properties of a metric space:** all distances are greater than or equal to 0, are commutative, and satisfy the triangle inequality

- **edge weight sum:** the sum of the edge weights in a path

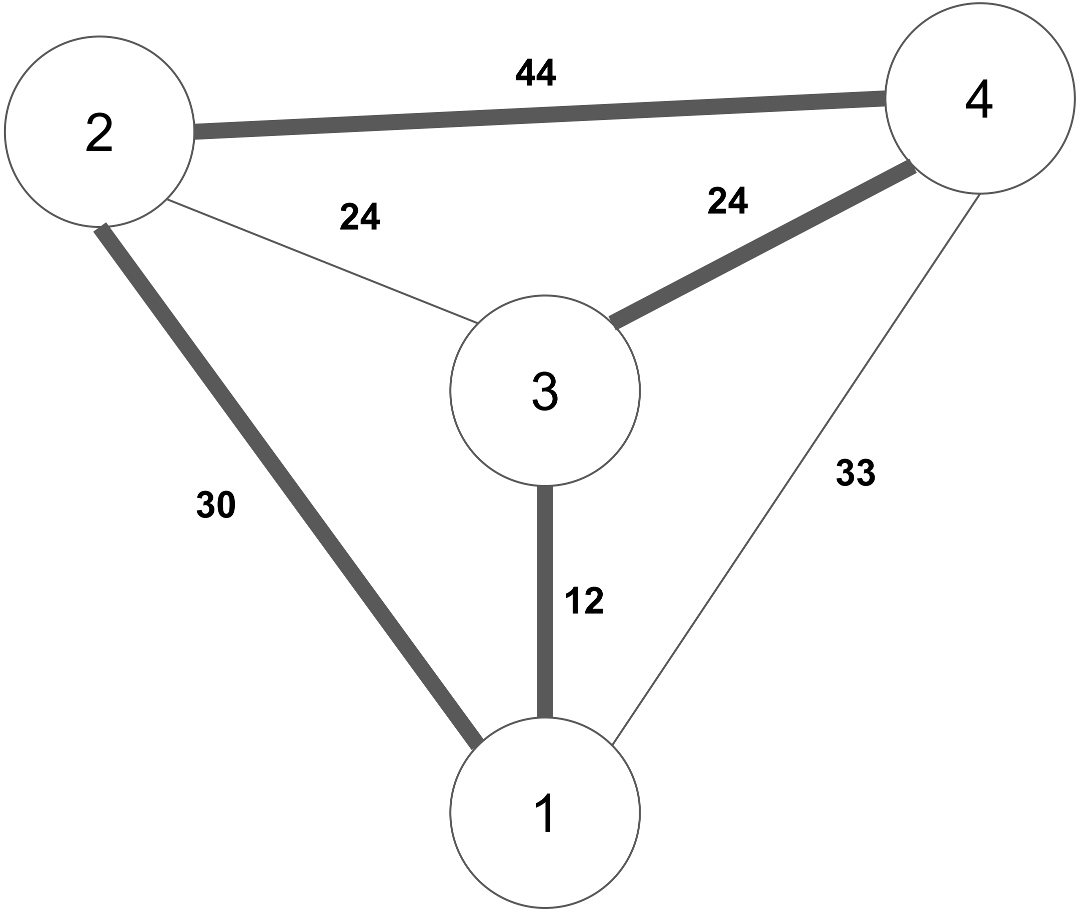
Example



On this **complete** and **metric graph.** Find a cycle with the **minimal edge weight sum.**

- Example of Metric TSP with four nodes
- **Complete** since every pair of nodes has an edge between them
- **Metric** since all distances are nonnegative and follow triangle inequality.

Example Solution



Brute-force Algorithm has time complexity of $O(n!)$

- standard Intel i7 process needs over 19 hours for a graph with only 17 nodes

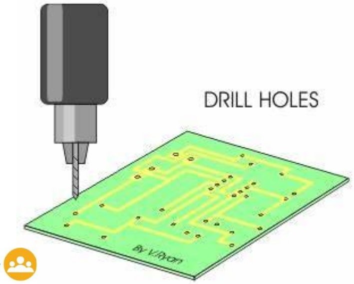
Best **Dynamic Programming** Algorithm has time complexity of $O(n^2 2^n)$

- standard Intel i7 process needs over 19 hours for a graph with only 38 nodes

Applications

Applications of Traveling Salesman Problem (TSP)

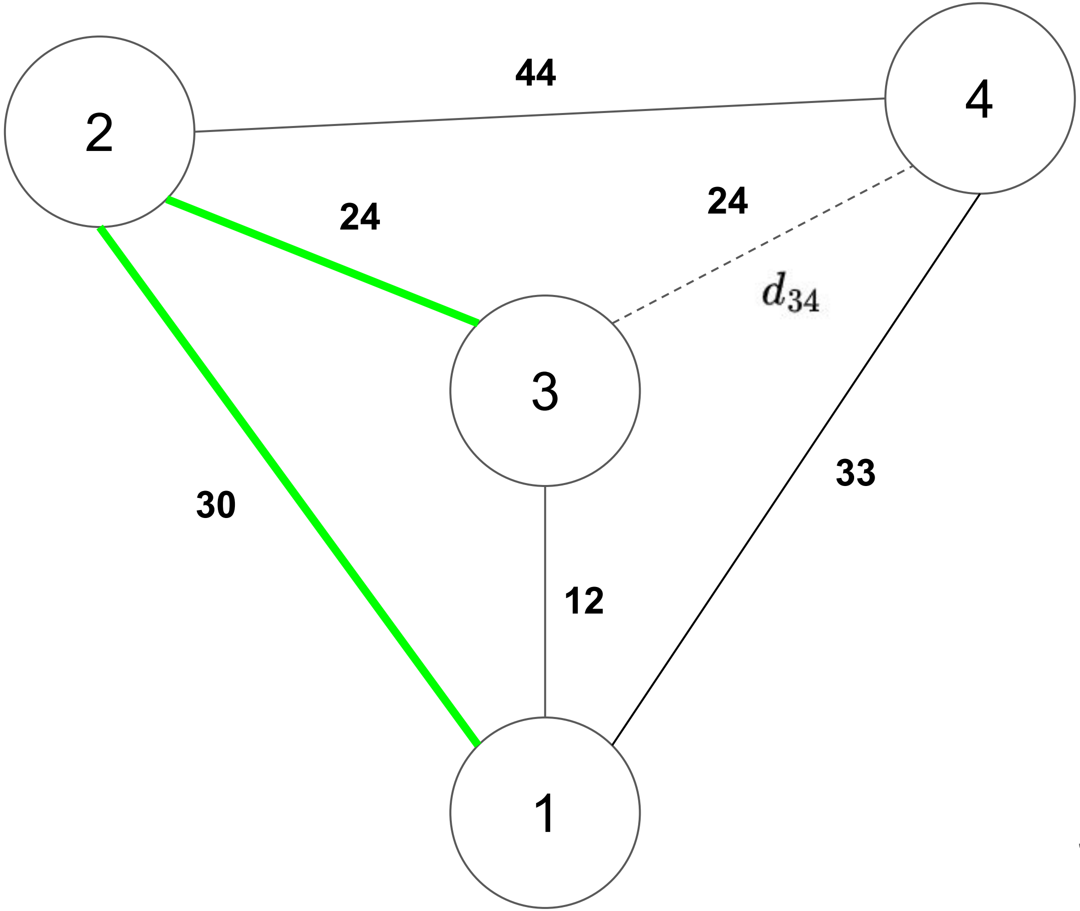
- TSP has countless applications across manufacturing, genomics and logistics.
  - orienting machine head when drilling holes in printed circuits
  - collecting goods from warehouses to transport to customers
  - positioning machinery in X-ray crystallography
  - connecting components in a computer board
  - overhauling gas turbine engines



- It has numerous other applications in various other sectors, and optimized algorithms could save countless hours and trillions of dollars

Best Classical Algorithm

Dynamic Programming Solution



Breaks the problem into $n2^n$ subproblems.

Link subproblems using the recursion $C(\{1\},1) = 0$ and

$$C(S,j) = \min_{i \in S : i \neq j} C(S - \{j\}, i) + d_{ij}$$

$C(S,j)$ represents the **minimal edge weight sum path** starting from **1** and ending at ***j*** of the subset ***S*** of nodes

d_{ij} is the edge weight connecting *i* to *j*

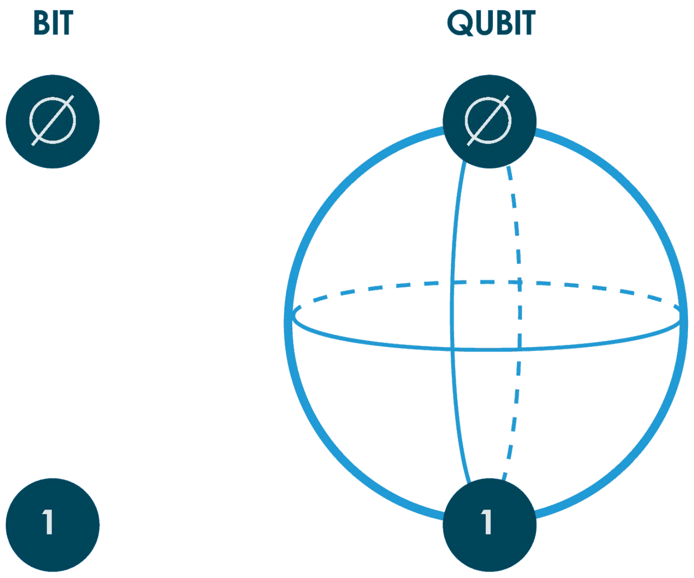
Ex.) $C(S - \{4\}, 3) + d_{34}$ on left

Stores sum in a memoization table.

Quantum Computing and the Power of Parallelism

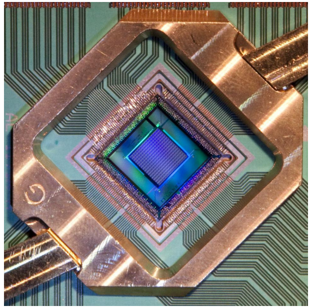
Quantum Computing

- Quantum Computing is a framework that exploits properties of quantum mechanics (superposition, entanglement, etc.) to speed-up computations
- Based on **quantum bits** or **qubits,** units that could take 0, 1 or a superposition of the two states.
- The first quantum computers were designed developed in 1980, and the first algorithm came in 1993.
- Has the potential to solve problems unsolvable for classical computers - **quantum supremacy**

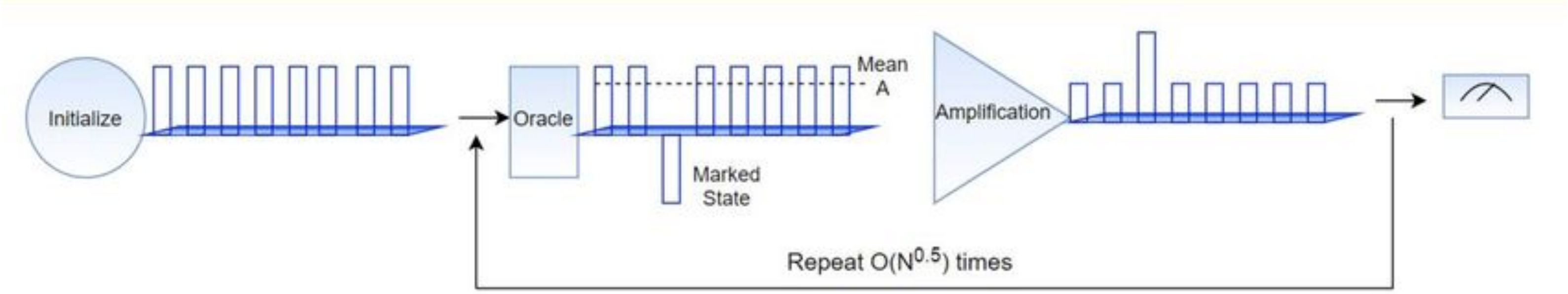


Noisy Intermediate Scale Quantum (NISQ) Era Technology

- Limited **Quantum Volume** - few high performing qubits with high error rates
- Large problems cannot be encoded and solved accurately
- For small inputs, computers can evaluate multiple inputs at once - **quantum parallelism**



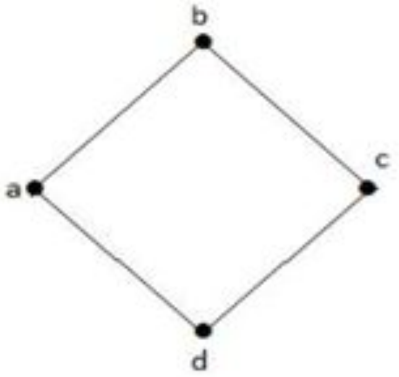
- **Grover's Algorithm:** one best current quantum algorithms, able to find a marked element in an array with size ***n*** in $O(\sqrt{n})$



Our Novel Work

Trimming the Graph

- To encode the problem on current quantum devices, we remove vertices not on the optimal subpaths
- Using the metric assumption, we mathematically prove the algorithm retains optimality
- This preprocessing step serves multiple purposes
  - ensures algorithm is adaptive, optimizing runtime based on input graph
  - decrease the classical portion of the algorithm geometrically, by reducing search space
  - reduces the input so it can be run on a Noisy Intermediate Scale Quantum (NISQ) device



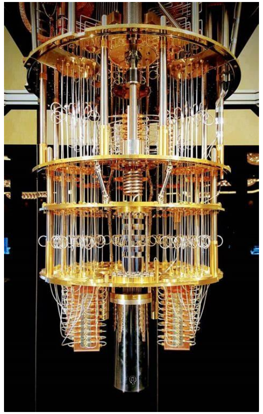
Our Adaptive Hybrid Algorithm

1. Apply **Johnson's Algorithm** to find shortest paths for subproblems

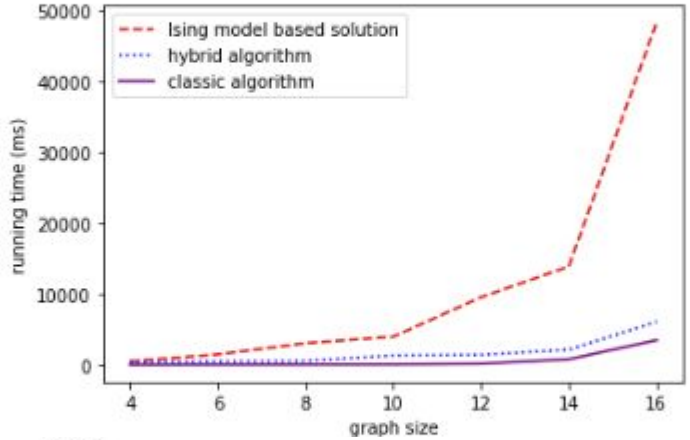
2. **Trim** the input graph to run on quantum computers

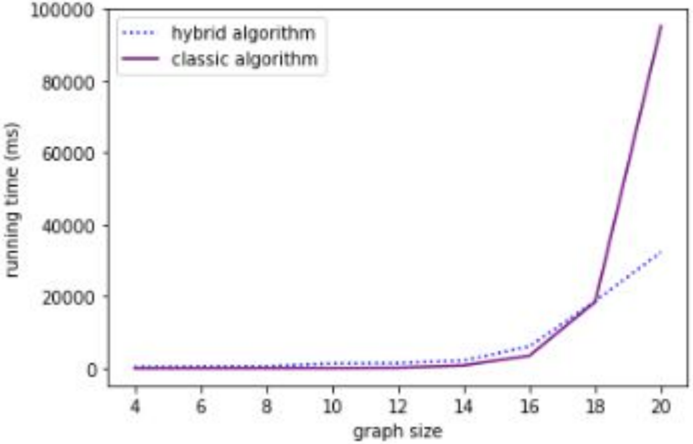
3. Search for the smallest path

- if encoded with less than 7 qubits use IBM **circuit-based** algorithm
- else use DWAVE **ising-model** based algorithm



Results and Conclusion





- Our novel hybrid work nearly matched the best classical algorithm for small inputs
- The hybrid algorithm outperformed the classical work for larger inputs and had a smaller rate of growth
  - hybrid grew by a factor of 5 while classical grew by factor of 24
  - had a lower time complexity
- Algorithmic performance will improve, as technology develops
- Approach can be applied to other NP-Hard Problems

Thanks for your Attention!
Questions?